EFFECTIVE INTEREST RATES AND BANK ADVERTS: MAKING SENSE OR MAKING CENTS?

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Effective interest is an important concept for personal finance. In this paper the concept is defined and illustrated by means of numerical comparisons with nominal rate. A derivation of the formula is provided and the concept is applied to advertised rates of bank products. It appears that these banks advertise average effective annual rates rather than the effective annual rate determined by the conventional formula.

INTRODUCTION

The concept of effective interest is important because it enables us to compare different interest rates over different periods, and with different compounding frequencies. Consequently it has important practical implications in the context of personal finance. In the school curriculum, effective interest is introduced in Grade 11 Mathematics and Mathematical Literacy but text books generally do not extend their discussion to engage with the relationship between effective rates and actual banking products. Based on personal experience and discussions with teachers, both teachers and learners find the concept of effective interest elusive and therefore difficult. However, there appears to be no research in this area, which is not surprising given the dearth of research into teaching and learning financial maths in general. In this paper I explore the notion of effective interest, by focusing on its definition, its connection to percentage increase, and the derivation of the effective interest formula. I then discuss the interest rates of notice deposits as advertised by two of the big South African banks and show how effective rates are used in each case.

Defining effective interest rate

An effective annual interest rate refers to the annual rate of interest of an investment when compounding occurs more often than once a year. It therefore takes into consideration the effect of compounding (Cussen, 2013; Investopedia, 2014). Definitions of effective interest typically refer also to nominal rate. For example, the following definition is provided in the National Curriculum Statement:

> Effective interest – the annual rate which is equivalent to a **nominal rate** when compounding is effected more often than once a year (e.g. 12% p.a. compounded monthly is equivalent to 12,68% p.a.; the nominal interest rate (i) is 0,12 and the effective interest rate is 0,1268).
>
> (Department of Education (DoE), 2003, p. 87, emphasis in original)

Many definitions provide a formula for calculating effective annual rate. In order to do so, it is first necessary to define some terms.

If i is the nominal interest rate per annum, then $i_m = \frac{i}{m}$ represents the effective rate per period, where m is the number of compounding periods in a year. Thus i_4 is an effective quarterly interest rate, i_{12} is an effective monthly interest rate, and i_1 is an annual interest rate with annual compounding. In this case the nominal and effective rates are the same. Based on these definitions of terms, we have the following formula for effective annual interest rate: $i_e = \left(1 + \frac{i}{m}\right)^m - 1$.

Making sense of effective interest through numerical examples

It is helpful to illustrate the notion of effective interest by means of examples, and in contrast to nominal interest. Consider the scenario where I invest R800 for a year at 6% p.a. compounded monthly. Each month I receive 0.5% interest on the latest balance. This means that each month I get more interest than the previous month because 0.5% is calculated on a slightly larger amount each time. By contrast, if we consider a simple interest scenario then I still get 0.5% each month but this is always calculated on the original balance of R800.

In Table 1 I compare these 2 scenarios. I include month zero to indicate the starting amount for the 12-month period. In the simple interest section I get R4 interest each month and this accumulates to R48 over the year. In the compound interest section, the interest amount increases each month, and the total interest is R49.34.

Simple interest			**Compound interest**		
End of month	**Balance**	**Interest**	**End of month**	**Balance**	**Interest**
0	800.00		0	800.00	
1	804.00	4.00	1	804.00	4.00
2	808.00	4.00	2	808.02	4.02
3	812.00	4.00	3	812.06	4.04
4	816.00	4.00	4	816.12	4.06
5	820.00	4.00	5	820.20	4.08
6	824.00	4.00	6	824.30	4.10
7	828.00	4.00	7	828.42	4.12
8	832.00	4.00	8	832.57	4.14
9	836.00	4.00	9	836.73	4.16
10	840.00	4.00	10	840.91	4.18
11	844.00	4.00	11	845.12	4.20
12	848.00	4.00	12	849.34	4.23

Table 1: Comparison of simple and compound interest each month for 12 months

I now show the connection between this accumulated amount, percentage increase and the effective annual rate. If we calculate the percentage increase on the principal amount in both cases we get:

$$\text{SI increase} = \frac{848-800}{800}\% = 6\% \qquad \text{CI increase} = \frac{849.34-800}{800}\% = 6.1675\%$$

It is important to note that the amount of R849.34 has been rounded to 2 decimal places. If we work to higher levels of accuracy, the percentage increase will be 6.167781% (rounded to 6 decimal places).

Now since the effective rate is an annual interest rate that will make R800 grow to R849.34 in one year, we use the compound interest formula as follows to determine the effective rate:

$$A = P(1+i)^n$$

$$849.34 = 800(1+i)^1$$

$$i = \frac{849.34}{800} - 1 = 1.06167781\ldots - 1 = 6.167781\%$$

This is the same answer as that obtained from the percentage increase calculation, which is not surprising since $i = \frac{849.34}{800} - 1$ is an equivalent form of that calculation. Thus we see that the effective annual rate is the same as percentage increase over a one year period.

Deriving a formula for effective rate

In many school texts the formula for effective rate is simply stated in a form similar to that given earlier in the paper. Consequently it may appear intimidating to learners. However, it may be less intimidating if learners are aware of its derivation which begins with equating two versions of the compound interest formula.

We begin with a general form of the compound interest formula with m compounding periods per year: $A_1 = P\left(1+\frac{i}{m}\right)^m$. When we work with effective rate, we compound once at the end of the year. So we have $A_2 = P\left(1+\frac{i_e}{1}\right)^1$. Now since the two calculations must produce the same outputs, we can equate the two equations: $P\left(1+\frac{i_e}{1}\right)^1 = P\left(1+\frac{i}{m}\right)^m$. Since P is common we get: $1+i_e = \left(1+\frac{i}{m}\right)^m$ which leads us to: $i_e = \left(1+\frac{i}{m}\right)^m - 1$. From the formula we can deduce that the effective rate is independent of the principal amount but dependent on the number of compounding periods. Testing our earlier scenario with R800 invested at 6% compounded monthly, we get $i_e = \left(1+\frac{0.06}{12}\right)^{12} - 1 = 6.16778118\ldots\%$ as expected.

Application to bank interest rates

In this section I consider two adverts for fixed period investments from two of the big South African banks. When banks advertise notice deposits, they are required to indicate whether the advertised rate is a nominal or an effective rate. They also generally offer different rates depending on the period of the investment, and may offer different rates depending on when interest is paid out.

For example, if interest is paid out at the end of the investment (i.e. on maturity), the rate indicated will be higher than if interest is paid out at regular intervals during the life of the investment.

Figure 1 provides nominal and effective rates for a fixed deposit account from First National Bank (First National Bank, 2014). According to the information provided, if an amount is invested for 12 months, the nominal rate is 5.9%, and the effective rate is 6.06%, based on the convention of monthly compounding.

Fixed Deposit Account

Published at 2014-03-04

Period	Interest (nominal)	Interest (effective)
1 month	4.00%	4.00%
2 months	4.80%	4.81%
3 months	5.10%	5.12%
4 months	5.10%	5.13%
5 months	5.10%	5.14%
6 months	5.60%	5.67%
7 months	5.60%	5.68%
8 months	5.60%	5.69%
9 months	5.60%	5.71%
10 months	5.60%	5.72%
11 months	5.60%	5.73%
12 months	5.90%	6.06%
13 months	5.90%	6.08%
18 months	6.20%	6.48%
24 months	6.50%	6.92%
36 months	7.00%	7.76%
48 months	7.25%	8.38%
60 months	7.65%	9.28%

Figure 1: FNB advertised rates for Fixed Deposit Account

We can confirm these figures by using the formula: $i_e = \left(1 + \frac{5.90}{12}\right)^{12} - 1 = 6.06\%$ (to 2 dp). However, if we try to confirm figures for periods shorter than or longer than one year, we may not get the effective rates advertised. For example, consider the case of 3 years: the bank quotes a nominal annual rate of 7% and an effective annual rate of 7.76%. We can check this by adapting the formula derived above. We know that 7% is the nominal rate that will be compounded 36 times over 3 years, so we have: $A_1 = P\left(1 + \frac{0.07}{12}\right)^{36}$. We need to determine an effective rate that will be compounded annually for 3 years to obtain the same final amount, so we have $A_2 = P(1 + i_e)^3$. Since $A_1 = A_2$ and the principal amounts are common, we have:

$(1 + i_e)^3 = \left(1 + \frac{0.07}{12}\right)^{36}$. Solving for i_e we get i_e= 7.23% (to 2 dp). This is the rate which is compounded annually for 3 years and which produces the same amount as 7% compounded monthly for 3 years. But this is not the effective rate quoted by the bank. There is, however, a second possibility for calculating an effective rate.

Here we think about an *average* rate over the 3 years. In other words we determine the total percentage increase over the 3 years and then divide this by 3. This is the same as working with simple interest. So we have A_1 as defined above giving us the total amount accumulated over 3 years. Then we use the simple interest formula over 3 years with an effective rate as follows: $A_2 = P(1 + 3 \cdot i_e)$. Once again, since $A_1 = A_2$ and the principal amounts are common, we have: $1 + 3 \cdot i_e = \left(1 + \frac{0.07}{12}\right)^{36}$. Solving for $i_e$we get i_e= 7.76% (to 2 dp). This is the rate quoted by the bank. So it appears that the bank is quoting average rates for the 3 year period. This rate is higher than the rate calculated previously option because it works off the principal amount rather than from the closing balance at the end of each year. It is not surprising that a bank would choose to quote a higher rate to draw clients. We can show that the same applies for the figures quoted for 6 months and 24 months.

Now consider Figure 2 where Nedbank offers the following rates on an investment product (Nedbank, 2014), and indicates different rates depending on when interest is paid out.

Nedbank Green Savings Bonds

18, 24, 36 or 60 month deposits, for clients less than 60 years of age.

Balance	Period of Investment	Interest Monthly	Interest Half-yearly	Interest on Expiry
R1 000 or more	18 months	6.61%	6.70%	6.92%
	24 months	7.20%	7.30%	7.71%
	36 months	7.73%	7.85%	8.66%
	60 months	8.36%	8.50%	10.33%

Figure 2: Nedbank advertised rates for investment product

If interest is paid out monthly, then we are working with a nominal annual rate since no interest accumulates in the account. If interest is paid on expiry, we are working with an effective annual rate. In other words, the interest rate on expiry should generate the same amount of interest as we would get by receiving interest monthly and withdrawing it from the account.

Once again we need to consider both methods of calculating the effective rate. For illustrative purposes I focus on 24 months and 60 months.

24 months

$$(1 + i_e)^2 = \left(1 + \frac{0.072}{12}\right)^{24}$$

i_e= 7.442417% to 6 dp

60 months

$$(1 + i_e)^5 = \left(1 + \frac{0.0836}{12}\right)^{60}$$

i_e= 8.687884% to 6 dp

Neither of these figures agrees with the advertised rates for interest paid on expiry.

Below I provide the calculations for an average effective rate:

$$1 + i_e \cdot 2 = \left(1 + \frac{i}{12}\right)^{2\times12}$$
$2i_e$ =0.15438729
i_e =7.719365% to 6 dp

$$1 + i_e \cdot 5 = \left(1 + \frac{i}{12}\right)^{5\times12}$$
$5i_e$ =0.516720888
i_e = 10.334418% to 6 dp

If we truncate the answers (to 2 decimal places), we get the quoted rates for interest paid on expiry. Once again, it appears that the bank is quoting average annual rates.

In the case where interest is paid half-yearly, it appears that the quoted rates have also been calculated using an average rate. I illustrate this for the case of 18 months by making the relevant substitutions:

$$1 + \frac{1}{2} i_e = \left(1 + \frac{i}{12}\right)^{0.5\times12}$$
$$\frac{1}{2} i_e = \left(1 + \frac{0.0661}{12}\right)^{6} - 1$$
$$\frac{1}{2} i_e = 0.033508 \ldots$$

So $i_e = 6.70\%,$ truncated to 2 decimal places as quoted in the advert. In the first line of the above equation, the right-hand side represents the accumulated amount after six months of monthly compounding. The left-hand side must therefore yield the same amount over a six-month period, using simple interest. This can be achieved by halving the annual interest rate which means we obtain 3.35% interest over six months, and this equates to an annual rate of 6.7%. Using the same calculation with the relevant monthly rate, we can obtain all the quoted rates for interest paid half-yearly.

CONCLUSION

In this paper I have attended to definitions of effective rate and to numerical illustrations that contrast nominal and effective rates. I have discussed the derivation of the formula, showing how it emerges from the compound interest formula, and have applied the formula to two investment products. Based on calculations I have shown that the banks are advertising *average* effective annual rates rather than the effective rate we obtain from the effective annual rate formula used in schools. This suggests it may be useful to teach learners about average effective rates in order to help them make sense of the banking products they will encounter later in their lives.

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